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Applying Political Apportionment to Pre-Allocate Float / Buffer Ownership

Yi Su and Gunnar Lucko, Ph.D.

Department of Civil Engineering, Catholic University of America, 620 Michigan Avenue NE, Washington, DC 20064, USA

Abstract

Float in project schedules can provide important protection for activities if delays occur. Traditionally, total float exists along noncritical paths and shared among these activities. This has given rise to the longstanding question of who owns float. However, by definition the critical path lacks the unique flexibility provided by float. This conundrum is newly addressed by explicitly assigning float from the project finish as a buffer. Another more specific question appears: If each critical activity is entitled to a portion, what should its share be to protect the project? And how well does it fulfill the criterion of fairness, since activities are typically performed by different companies? Political apportionment is an established area to fairly distribute parliamentary seats by the votes that representatives have received. It provides systematic quota calculation with rounding methods to give consistent integer results that consider the relative size, i.e. importance, of competing participants. Inspired by apportionment, this research adapts a methodology to solve the problem of optimizing float allocation. Its contribution to the body of knowledge is twofold: First, float allocation is integrated with various apportionment methods. Second, an index is created to compare their performance based on Monte Carlo simulation. This research thus transforms float from being a passive byproduct of schedules to an active asset with which project managers can reduce delays. It builds a foundation for future research on active float management.

Keywords: Critical path, float ownership, political apportionment, project float, schedule activities.

1. Introduction

Time performance is a crucial aspect in project management. Construction delay, which is defined as “an act or event that extends the time required to perform tasks under a contract” (Stumpf 2000, p. 32), is the main contributor to bad time performance suffering the construction industry (Birgonul *et al.* 2015). For example, Mahamid *et al.* (2012) studied road construction firms in Palestine. The results showed that 75% of contractors had experienced 10-30% as a delay magnitude, 20% even indicated that their delay magnitude was 30-50%. A statistical analysis of road projects in Indiana found 1655 of 1862 (89%) delayed (Bhargava *et al.* 2010). Menches and Hanna (2006) studied electrical projects in the U.S. Among 60 responses, 53 projects (88%) were delayed. While an even more abundant literature exists that contain similar alarming delay statistics, for brevity only a selection of them has been listed here.

Reasons for poor time performance are twofold: 1. An as-planned schedule that is too tight; or 2. the as-built time performance is indeed poor. The former is beyond the scope of this paper and will be covered by future research. But for the latter, buffers and total float (TF) are the two basic protection mechanisms that provide “a cushion or shield against the negative impact of disruptions and variability” (Russell *et al.* 2014, p. 2). Scholars of the critical path method (CPM) have emphasized the classic ‘*who owns float*’ problem, where TF “represents the length of time an activity’s finish date may be delayed without affecting the completion date of the entire project” (de la Garza and Vorster 1990, p. 716). “Total float is a by-product of critical-path-method calculations” and should be traded as a commodity between project participants (*ibid.*). Time performance is directly related to contractual reward / penalty schemes, i.e. bonuses or liquidated damages. Thus how to fairly allocate such float is important for all participants.

* Corresponding author. Tel.: 001-202-319-4381, fax: 001-202-319-6677.
E-mail address: lucko@cua.edu.

2. Literature Review

2.1. Float / Buffer Allocation

Float and buffers “are two closely related phenomena that only differ in whether they are intentional” (Lucko *et al.* 2016, p. 2). Both of them can be used to protect the project finish from delays. Yet activities in a project schedule are conducted by various participants (owner, general contractor (GC), or subcontractors). “The complexity of distributing the float among these parties stems from the fact that the TF belongs to the path, not to the activity, and there can be many project parties on the same path” (Al-Gahtani 2009, p. 88). So does the buffer. Previous research can be grouped as participant-oriented versus activity-oriented. The former deals with float in terms of whether the owner or GC owns it, and is “based on the basic concept that the party who has the greatest risk in a project should be entitled to float ownership” (Al-Gahtani 2009, p. 88). But its problem is that a contractual risk factor is defined as a constant percentage, but it “should be changed for each activity based on the contract terms” (*ibid.*, p. 92). As a result, a solution for participant ownership would still have to subdivide contract risk factors by what actual work they self-perform (owner) or perform (GC). For activity-oriented, Pasiphol (1994, p. 94) advocated to “distribute more total float to activities requiring more time to be completed”. But such research was based upon a fixed as-planned schedule without considering any probabilistically distributed durations, and its concept ‘float in proportion to duration’ would require mathematical analysis of its fairness. Inspiration is provided by political apportionment in Federal systems: For the example of the E.U. (Kirsch 2007, Pöppe 2007), it was proven that assigning voting weight in proportion to the square root of the population n of each country will strike a perfect balance between ‘one vote per country’ that is proportional to n^0 versus ‘one vote per person’ that is proportional to n^1 . An opportunity exists to adapt this successful approach to solve how to fairly allocate a buffer or float specifically to the critical activities.

Critical Chain Project Management (CCPM) was developed to ensure a timely project finish by inserting limited buffers and imposing a general sense of urgency. It protects the time performance of critical activities with a single project buffer (PB) at the end of the critical chain. It analogously protects noncritical activities with feeding buffers (FB) at the ends of non-critical chains (Herroelen and Leus 2001), i.e. merges of side paths. If an activity is delayed, the FB will act as a local cushion or the PB as a global cushion. “But this approach radically cuts activity durations, assuming that they harbor individual float, returns half of it as a single large end buffer [PB], and then accepts that said ‘block’ is consumed fully without tracking by whom” (Lucko and Su 2016, p. 1). The CCPM has drawbacks, notably that (a) aggressively cutting their initially planned durations in half, then aggregating half of the summed cuts back into PB (Goldratt 1997) is regrettably not founded on any scientific approach or real-world data, (b) while it seeks to protect its critical chain (akin to critical path), it does not explicitly allocate PB, which can cause legal disputes over delay claims, and (c) it is fundamentally unfair, because it espouses a ‘first-come, first-serve’ tactic.

Table 1 compares float / buffer ownership approaches from the literature (e.g. Al-Gahtani 2006, Al-Gahtani and Mohan 2007, Arditi and Pattanakitchamroon 2006, Prateapusanond 2003, Pasiphol 1994, de la Garza *et al.* 1991, Householder and Rutland 1990, Ponce de Leon 1986). Problematically, they either contradict each other (Al-Gahtani 2009), (owner versus contractor approach), or leave unclear how to fair split should be achieved (project approach).

While some approaches suffer from an unclear split between owner and GC, others are activity-oriented and focus on subcontractors who perform the actual work. Some require *ex post* analysis (bar and day-by-day approaches). But only the total risk approach provides an explicit percentage of TF ownership *a priori*. Such PB/FB or TF approaches have a similar time-related object – buffer or float. As has been mentioned before, their difference is that the former is intentionally imposed by the CCPM scheduler, while the latter is calculated as an artifact of the activity durations and links in the schedule. But except for CCPM, which has the aforementioned drawbacks, all of these ownership approaches can only handle TF and do not protect the critical path, which by definition urgently lacks protection.

Yet extra ‘contract float’ (CF) exists between the as-planned project finish and the contract due date (O’Brien and Plotnick 2009, Keane and Caletka 2008). This paper assumes that uninflated activity durations that are based purely on a realistic productivity are known to calculate the ‘raw’ project finish. This CF is the only time contingency that is available specifically to critical activities. But it currently lacks an explicit allocation method. Therefore this research poses a question: *How should CF be used optimally to protect the critical path, i.e. how should it be allocated fairly?*

Table 1: Comparison of Float / Buffer Ownership Approaches

Approach	Critical	Non-Critical	Participant	Allocation
Owner Ownership	N/A	TF	Owner	100%
Contractor Ownership	N/A	TF	GC	100%
Project Approach	N/A	TF	Owner / GC	x% / x% *
Equal Proportion	N/A	TF	Owner / GC	50% / 50%
Bar Approach	N/A	TF	Activity	Consume TF, then excuse
Contract Assigns Risk	N/A	TF	Owner / GC	x% / x% *
Path Distribution	N/A	TF	Activity	Based on duration
Day-by-Day Approach	N/A	TF	Activity	Track TF changes
Commodity Approach	N/A	TF	GC	Trade TF for cash
Total Risk Approach	N/A	TF	Activity	TF proportion to risk
CCPM Method	PB	FB	Activity	First-come, first-serve

* Undefined in case law. Note: PB (FB) at end of (non)critical chain; TF shared along noncritical path.

2.2. Political Apportionment

Political apportionment deals with the issue that “electoral votes must be translated into specific seat allocations... [t]he seat allocations are of course integer numbers” (Schuster *et al.* 2003, p. 652). Suppose P total votes are given to n parties, with p_i votes for the i^{th} party, where $\sum p_i = P$. A total of N seats must be apportioned to these parties. Thus the party percentage is p_i / P . The exact allocation (often called fair share or quota) is $q_i = N \cdot p_i / P$. But this value is not always the required integer, as fractional seats are impossible. Calculating seats from votes thus is adjusted, for which various apportionments have emerged, including largest remainder (LR) and divisor methods (e.g. Jefferson, Adams, Huntington-Hill, etc.). The LR first rounds quotas down to the nearest integer ($\lfloor q_i \rfloor$). The difference between their sum $\sum \lfloor q_i \rfloor$ and N are the leftover seats. Next it takes the party with the largest remainder, $\max(q_i - \lfloor q_i \rfloor)$, and assigns one leftover seat. If seats are still left, assigns one to the second largest remainder, and so forth. But LR suffers from the so-called ‘Alabama Paradox’ that *increasing* N by a day may counterintuitively cause the smallest party to suddenly *lose* a seat, because rounding the revised values may give a slightly different integer allocation to all parties (Wiseman 2001). The latter includes several methods that seek a proper divisor λ . The Greatest Divisor (GD) and Smallest Divisor (SD) methods differ only in that the former rounds down the quota divided by λ , but the latter rounds up. Geometric Mean (GM), Harmonic Mean (HM), and Arithmetic Mean (AM) use different means: Both GM and HM round down quotas $f_i = \lfloor q_i \rfloor$ and calculate the mean of f_i and $f_i + 1$; AM takes the arithmetic mean, but different from the other two mean methods it rounds to the nearest integer, instead of always rounding down. Then GM and HM compare the quota with the mean to judge if f_i and $f_i + 1$ should be used; AM just rounds the operand to the nearest integer. Note that the quota q_i in GM and HM may not yet equal N . In such case, q_i is adjusted through a divisor. Divisor methods require time-consuming iterations that can be automated with computer programming.

Analogies exist between political apportionment and float / buffer pre-allocation (Lucko and Su 2016): 1. The goal is to fairly apportion an limited asset; 2. seats or float / buffer represent a power of entitlement (party in politics, activity in schedule); 3. results must be integers (seats or time unit like workdays) whose sum must equal the total amount that is available. Therefore it is feasible to newly apply political apportionment methods to float / buffer pre-allocation. Two **Research Objectives** are set to address the **Research Question** of optimally pre-allocating the CF:

- Inspired by political apportionment, create a float pre-allocation method whose quota is based upon activity duration with variable exponents to evaluate the protective ability in proportion to n^0 , the square root $n^{0.5}$, and n^1 .
- Create a measurement index to evaluate the performance of Monte Carlo simulations of the LR and AM methods.

3. Methodology

Applying political apportionment to pre-allocating float / buffer to each critical activity has these steps: First, inputs of activity name, duration D_i , and sequence are used in CPM to identify critical activities. Second, calculate D_i to the power of n akin to ‘votes’ (p_i) for each critical activity. Third, incrementally increase the float / buffer akin to ‘total seats’ (N) and select an apportionment method. The output is an allocation of float / buffer like apportioning seats to each party (s_i). Fourth, run a Monte Carlo simulation L times to generate randomized as-built schedules with delays from given probability distributions for each activity. Fifth, calculate any overrun of the fixed duration D_i as an integer time, e.g. $\lceil d_i - D_i \rceil$ and compare it with the allocated float (s_i). Sixth, for each critical

activity calculate the discrete count of overruns out of L times, and the overrun period $(d_i - D_i) - s_i$. Seventh, plot the simulation results of discrete count and overrun period over the incrementally increased CF . They are expected to be decreasing stepped 2D profiles of counts over time (discrete performance) and period over time (continuous performance). Eighth, calculate statistical indices for each profile to quantify their protective ability against delays: The arithmetic mean for each profile between the times when the profile begins to decrease and when it reaches zero (and is thus saturated with float so that delays can be fully mitigated). Then calculate minimum, mean, maximum, and standard deviation of these averages. Finally, after running simulations for all apportionment methods and all exponents, select the best combination of method and n , which should have the smallest mean and standard deviation for its discrete and continuous performances. This new methodology has integrated the float pre-allocation with political apportionment and fulfills **Research Objective 1**. Critical path changes are not examined, but will be addressed by future research.

4. Validation

A validation schedule is selected from the Project Scheduling Problem Library (PSPLIB, Kolisch and Sprecher 1996). It has 30 activities plus one dummy start and finish. The authors have developed a computer program that can automatically visualize the network per Figure 1. Since the PSPLIB data only include the sequence of activities and fixed durations, this paper adds a probability distribution for activity durations. Users can specify any type of distribution. Here, 90% and 150% of the fixed mode is used as the lower and upper limits of a triangular distribution. Note that fractional overruns are rounded up to count them as integer days, the basic time unit that is assumed herein. While this example only contains Finish-to-Start links, the computer program and method can handle any link type.

The optimistic, most likely (mode), and pessimistic project durations are 34.2, 38, and 57. Theoretically the most float that should be required is the difference between the pessimistic and the optimistic duration, which is 22.8 days.

4.1. Simulation Outputs

Figures 2 and 3 present the count and period profiles for the three exponent cases with two methods (LR and AM) after 100 simulation runs. Note that all profiles in the count figures start below 100 counts, because some randomly sampled durations from the triangular distribution are shorter than the mode, so that no overrun occurs. As expected, all profiles decrease with increasing CF – the more float is allocated, the more each activity receives and the less often they overrun. The upper three plots in Figure 2 are the count profiles for exponents 0.0, 0.5, and 1.0 and LR, while the lower ones are the period profiles. In terms of the maximum of the averages of decreasing range, the $n = 1.0$ case provides the optimum (least) value of 13, compared to 19 and 15.5, respectively. But in terms of the mean of the averages, $n = 0.5$ is the best (2.84, compared to 3.05 and 3.10). Thus the group trend shows that the square root approach is better. Interestingly the Alabama Paradox is seen in the dashed circles when CF changes from 18 to 19 for $n = 1.0$; a peak shows how a short activity briefly receives less float, is more vulnerable, and may incur overruns.

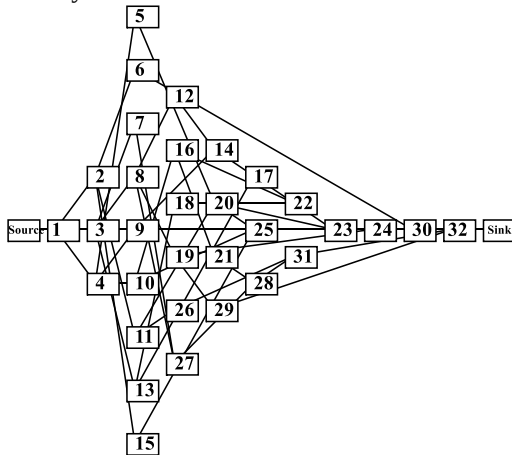


Figure 1: Network of the J30_1 Schedule

Figure 3 shows the count and period plots for AM with the same exponents. Characteristics similar to LR are seen. The Alabama Paradox disappears, because of the different apportionment; AM requires searching for a proper divisor iteratively. But different from political voting, some critical activities may have exactly the same mode duration and thus share the same quota. If so, the divisor method cannot provide a solution for certain CF values. Two adjustments are therefore made to gain a continuous profile:

- An increment (0.001) is added to (or deducted from) activities with the same initial duration, so that they have different quotas. It does not alter the solution, as it does not affect the duration structure within the schedule, but distinguishes their power;
- If no optimal solution can be found for a specific CF , then the solution from the previous smaller CF is kept. This ensures that at least such a suboptimum solution is provided as valid output.

Comparing the statistics for the outputs of Figure 3, the group trend again indicates that $n = 0.5$ is the most efficient way (the mean of averages is 2.74, compared to 2.90 and 3.06) to pre-allocate float to protect the critical

path from delays. Table 3 lists all results numerically (the minima are omitted for brevity). Its best combination is highlighted in bold. Interestingly, the results for $n = 0.5$ and $n = 1.0$ tend to be closer together. This is because pre-allocating the same float ($n = 0.0$) ignores any differences among activities. The best combination is AM and $n = 0.5$.

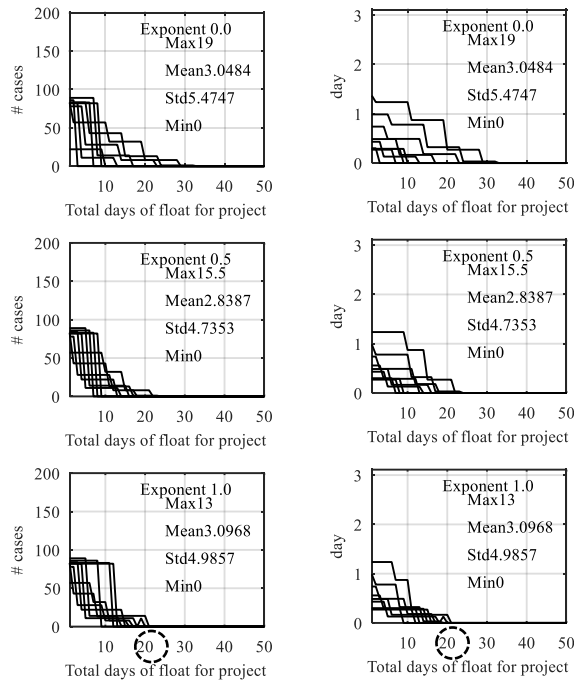


Figure 2: Count and Period Plots of Largest Remainder Method Simulation

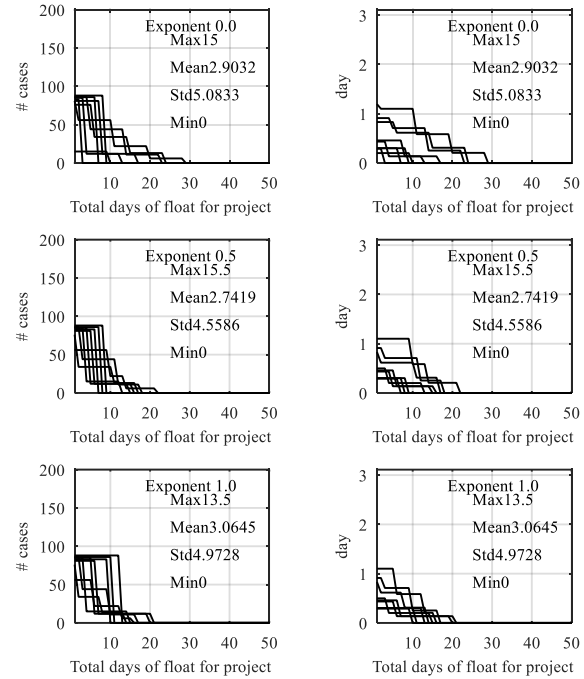


Figure 3: Count and Period Plot of Arithmetic Mean Method Simulation

4.2. Discussion

Comparing the statistical metrics of Table 3 has revealed that exponent $n = 0.5$ of the mode duration and the AM method should be selected when using the quota to determine the optimum float pre-allocation. In this combination, the apportionment method gives the smallest mean of the averages. This means that in most cases of the simulation the critical activities are ideally protected by their allocated integer part of CF . Both counts and periods of overrun confirm this observation. Theoretically, the method of using $n = 0.5$ should be fairer than 0.0 and 1.0. Exponent 0.0 unfairly favors short activities, because all critical activities have the same quota. Here short ones stop overrunning quickly and any extra allocated float is wasted, because their need is also small. Conversely, only a high value of CF would eventually saturated long ones. Such behavior relies on the assumption that risk is approximately proportional to duration, which has been used as a proxy for it. If shorter activities are riskier for some reason (Al-Gahtani 2006), then the model should not use duration, as has been described here, but a direct numerical value of risk for its quotas.

Table 3: Comparison of Apportionment Methods

Exponent	0.0			0.5			1.0		
Method	Max	Mean	Std.	Max	Mean	Std.	Max	Mean	Std.
Largest Remainder	19	3.0484	5.4747	15.5	2.8387	4.7353	13	3.0968	4.9857
Arithmetic Mean	15	2.9032	5.0833	15.5	2.7419	4.5586	13.5	3.0645	4.9728

The opposite happens for exponent 1.0, which strongly (directly proportionally) favors long activities. Any short ones are deprived of power to receive enough float and some may even never receive any. Note that the difference between durations among critical activities is a non-negligible factor, because it strongly affects how quotas are distributed. For example, a group of critical activities with very close durations $\{1, 2, 3, 4, 5\}$, and another with very diverse durations $\{1, 1, 1, 2, 10\}$ might perform differently for $n = 0.5$. This exponent is not always the best option, but each specific schedule may have a customized optimum exponent. This notion is beyond the scope of this paper and is left for future research. Comparing the simulated performance of LR and AM fulfills **Research Objective 2**.

5. Conclusions

Existing float ownership literature has unfortunately only focused on distributing TF, while CCPM is allocating buffers to provide a time cushion at the end of each chain without any scientific underpinning. A method to explicitly pre-allocated CF to the critical path has been lacking. This paper therefore has provided a detailed new methodology that integrates political apportionment methods into float allocation to answer ‘*who should own the float and how much to given to each critical activity?*’ Steps have included: 1. The CPM calculation of a fixed duration schedule to determine critical activities; 2. calculating a quota for each critical activity with different exponents (0.0, 0.5, and 1.0) and using various apportionment methods to obtain the float pre-allocation; 3. running Monte Carlo simulations to generate randomized schedules with a given probability distribution and comparing if the delay of critical activities overrun their allocated float or not by recording both overrun counts and periods. 4. plotting these results over the increasing amount of float, and determining the minimum, mean, maximum, and standard deviation of the average time when the profiles degrease. Validation by comparing results from simulating a PSPLIB schedule has shown that this method can select an ideal apportionment method and its exponent to provide optimum protection.

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